CSCI653 Assignment 2—Fast Multipole Method
Due: 11:59 pm, Monday, September 16, 2019

The purpose of this assignment is to acquire a good sense of computational complexity and floating-point performance, which are central to high performance computing (HPC), using the N-body problem and fast multipole method (FMM) [Journal of Computational Physics 73, 325 (1987)] as an example. Use the program fmm2d.c (http://cacs.usc.edu/education/cs653/src/FMM), which evaluates the electrostatic potential at all particles’ positions for a 2-dimensional system of charged particles using both $O(N)$ FMM and $O(N^2)$ direct sum. For this assignment, you can use any computer, which has a C compiler.

1. **Computational complexity**: Perform a series of runs of the original fmm2d.c, in which the problem size (i.e., the number of particles) is changed systematically: $N_{\text{par}} = 1,000 \times 4^{L-4}$, where the quadtree height $L = 4, 5, 6, 7$. Plot the elapsed wall-clock time $T$ as a function of $N_{\text{par}}$. Fit the measured $T$ to the formula, $T = C \times N_{\text{par}}^p$, and find the power $p$ for both FMM and direct calculations ($C$ is the other fitting parameter). Explain the observed power in the light of computational complexity.

   *Submit the plot, fitted power and your explanation.*

2. **Flop/s Performance**: Performance of a program is often measured in Gflop/s (gigaflop/s = billion floating-point operations per second). Modify fmm2d.c so as to count the number of floating-point operations (+, −, *, /) executed. Though it is difficult find how many operations are performed within math-library functions, for simplicity, here let us count sqrt(), log() and atan() functions as $FSQRT = 10$, $FLOG = 10$ and $FATAN = 10$ operations. Run the modified program for $L = 6$ and $N_{\text{par}} = 1,000 \times 4^{L-4} = 16,000$. Divide the resulting number by the elapsed time (in seconds), then divide that number further by $10^9$, to obtain the program’s Gflop/s performance.

   *Submit the modified program and the measured Gflop/s performance along with the machine information (processor, clock speed, etc.).*