GPU Accelerated Direct Kinetic Simulation Code for Collisionless Plasma Expansion

Yuan Hu

Department of Astronautical Engineering
University of Southern California
Los Angeles, CA, USA
yuanhu@usc.edu

Abstract—Collisionless plasma expansion is a fundamental physics problem in plasma science and has great impacts on engineering applications, such as fusion and electric propulsion. Though much more computationally expensive, the kinetic approaches are required for both electrons and ions in order to accurately solve the collisionless plasma problems. The grid-based direct kinetic simulation code with GPU acceleration is developed. We take advantage of this GPU accelerated direct kinetic code to study the Vlasov equation with 1D1V, 2D2V and 2D3V. The simulation results can be used to analyse the non-equilibrium effects on collisionless plasma expansions as well as to study the validity of numerous simplified models in the literatures.

Keywords—collisionless plasma expansion; direct kinetic method; GPU

I. INTRODUCTION

Collisionless plasma expansion is a fundamental physics problem in plasma science and has great impacts on engineering applications, such as fusion and electric propulsion. The collisionless plasma dynamics should be described kinetically by the Vlasov equation,

$$\frac{\partial f_\alpha}{\partial t} + \mathbf{v} \cdot \nabla_x f_\alpha + \frac{\mathbf{F}}{m_\alpha} \cdot \nabla_v f_\alpha = 0$$  \hspace{1cm} (1)

where $f_\alpha(t, x, v)$ and $m_\alpha$ is the particle distribution function and mass for species $\alpha$, respectively; $\mathbf{F}(t, x)$ is the force exerted on a particle at the position $x$. Usually, only electrostatic force is considered in $\mathbf{F}$ for the collisionless plasma expansion problem, namely

$$\mathbf{F} = q_\alpha \mathbf{E} = -q_\alpha \nabla \Phi$$ \hspace{1cm} (2)

The electric potential is solved from the Poisson’s equation,

$$\nabla \cdot (\epsilon_0 \nabla \Phi) = e(n_e - Z_i n_i)$$  \hspace{1cm} (3)

where $\epsilon_0$ is the permittivity of the vacuum, $e$ is the unit charge, $n_e(x)$ and $n_i(x)$ are number density of electron and ion, respectively, and $Z_i$ is number of charge one ion carries. $n_e$ and $n_i$ can be obtained by integrating $f_{e,i}$ in the velocity space,

$$n_{e,i} = \int f_{e,i} \, d\mathbf{v}$$ \hspace{1cm} (4)

Eqs. (1) - (4) constitute the Vlasov-Poisson (VP) system, which is the governing system of equations for the collisionless plasma expansion problems. Analytical solutions of VP system exist only in very limited academic cases, e.g. quasineutral approximations [1], [2], due to its nonlinear feature. Therefore, numerical simulations are required for investigating realistic physics phenomena.

The most widely used approach to solving the VP system is the particle-in-cell (PIC) method [3]. The plasmas (ions and electrons) are represented by a certain number of macro-particles, whose trajectories follow the characteristic curves of the Vlasov equation (1). The electric field, used to update the velocity of macro-particles, is solved self-consistently on a mesh discretized in the physical space. On the other hand, the Eulerian methods by directly discretizing the Vlasov equation on a grid of the entire phase space have received substantial attention recently [4], [5], [6], [7], [8], [9], [10], [11].

The main advantage of PIC method is that the macro-particles naturally carry the three dimensional information in both physical and velocity space. This makes PIC codes relatively cheap and easily extendable to high dimensional applications. The main disadvantage of PIC method is the statistical noise intrinsically accompanying the finite particle number methods, which makes it very difficult or prohibitively expensive to get satisfactory local properties. This problem is even worse when we deal with the collisionless plasma expansions, where the density difference across the domain of interest could be more than 10 order of magnitude because vacuum or near vacuum regions can exist. As an alternative, the grid-based or Eulerian methods are very suitable to overcome lack of precise description in low density region. During the past decade, the grid-based direct Vlasov simulation methods for problems of 2D(1D1V), 4D(2D2V) and 5D(2D3V) in phase space have been developed.

As can be seen from the distribution function $f(t, x, v)$ and the Vlasov equation (1), the requirements of memory and computation increase exponentially with the dimension. The distributed parallel computing technique is highly demanded for the grid-based kinetic methods, especially for high dimensional applications. Moreover, the extremely intensive computation associated with the direct Vlasov solvers makes it suitable to utilize GPU acceleration. Therefore, the ultimate goal of this project is to fully take advantage of the available heterogeneous parallel systems.
with GPU accelerators and to develop a grid-based Vlasov-Poisson solver aiming to handle high dimensional collisionless plasma expansions.

II. SIMULATION METHODS

We have developed a sequential 1D1V grid-based direct kinetic solver which employs the positive and flux conservative (PFC) method developed by Filbet et al. [5]. The PFC scheme is a conservative scheme and ensures the preservation of positivity of the distribution function. These advantages enable it to accurately handle strong nonlinear problems without unphysical instabilities. Moreover, its local reconstruction feature makes it well suited for parallel computing, especially for the GPU acceleration.

III. EXPECTED RESULTS

In principle, kinetic approaches are required for both electrons and ions due to the non-equilibrium nature in collisionless plasma modeling. However, electrons are usually handled by simplified models in practice due to the complexity of fully kinetic treatment of electrons. For example, the so-called Boltzmann relation for electrons

\[ n_e = n_{e0} \exp\left( \frac{e\Phi}{kT_{e0}} \right) \]  

is used ubiquitously [12], [13], [14], [15], [16], sometimes without any justification. In Eq. (5), \( n_e \) is the number density of electron, \( \Phi \) is the potential, \( n_{e0} \) and \( T_{e0} \) is the number density and temperature of electron at a reference location, respectively. Strictly speaking, the Boltzmann relation is valid only when the distribution function is always Maxwellian with a constant temperature \( T_{e0} \). Studies, including our PIC simulations [17], have revealed that the assumption of the Boltzmann electrons may fail at some point for the collisionless plasma expansions when the nonequilibrium effects or the heat transfer of electrons are important. The inherent statistical noise makes it difficult for the PIC simulations to accurately predict electrons’ behaviours in low density regions as well as local properties of the electrons. The development of grid-based Vlasov solver, with the help of the modern heterogeneous parallel computing system and GPU acceleration, will enable us to perform the most advanced fully kinetic study on this set of critical fundamental problems in plasma dynamics.

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REFERENCES


